Lehigh Conference On Differential Geometry
In honor of David Johnson's 70th birthday

October 14–16, 2022

Abstracts

All talks are held in Auditorium Room **#184 of Rauch Business Center** (621 Taylor St)
Campus map: https://www.lehigh.edu/∼inis/pdf/about/LUPackerCampusMap.pdf

Speaker: Vincent Borrelli (Université Claude Bernard Lyon 1)

Title: *Hyperbolic Planes in E^3 and their Limit Sets*

Abstract. In this talk, we will build an embedding f of a closed disk into a 3-Euclidean ball whose restriction to the interior is a C^1 embedding of the Poincaré disk and which is globally β-Hölder for any $0 < \beta < 1$. In particular, the limit set $L(f)$ of this C^1 embedding of the hyperbolic plane is a closed embedded curve of Hausdorff dimension one.

Speaker: Frank Morgan (Williams College)

Title: *Isoperimetric Theorems from David Johnson until Now*

Abstract. My paper with David Johnson on isoperimetric theorems is one of my most cited papers. The isoperimetric problem seeks the least-perimeter way to enclose a given volume. The paper gives a proof in any smooth compact Riemannian manifold that isoperimetric regions of small volume are nearly round balls. It also proves a generalized Cartan-Hadamard Conjecture for small volumes. It preceded the double bubble theorems, which provide the least-perimeter way to enclose and separate two prescribed volumes. Such results were recently extended by Milman and Neeman to multiple bubble theorems in Gauss space (2022, Annals of Math.) and more recently in Euclidean space and spheres. Many questions remain open.

Speaker: Oscar Perdomo (Central Connecticut State University)

Title: *Spectrum of the Laplacian and Stability Operator for CMC Hypersurfaces with two Principal Curvatures on the Spheres*

Abstract. In the first part of this talk we explain how to construct every cmc hypersurface with two principal curvatures on any space form endowed with a semi-riemannian metric. In the second part we consider the case when the ambient space is the $n + 1$ dimensional sphere and we explain how to compute the spectra of the Laplacian and the stability operator in terms of the eigenvalues of a second order Hill's equation. In particular, we show that every minimal non Clifford example has at least two eigenvalues of the Laplacian smaller than n. Continuing with the study of the spectrum of the stability operator, in the third part of this talk we explain a result with David Johnson on the nullity of minimal tori in the three dimensional sphere.

Speaker: Christina Sormani (CUNYGC and Lehman College)

Title: *Thoughts on Spacetime Intrinsic Flat Convergence*

Abstract. In order to define a spacetime intrinsic flat convergence, Carlos Vega and I defined the null distance to convert spacetimes endowed with regular cosmological times into metric spaces.

Anna Sakovich and I have proven that one can recover the causal structure from the null distance and the cosmological time. We apply this to show that a distance-preserving time-preserving bijection between the spacetimes endowed with a null distance is in fact a Lorentzian isometry under suitable conditions. Next we will prove there are biLipschitz charts so that we may view the spacetimes endowed with the null distance as integral current spaces. This will allow us to rigorously define the spacetime intrinsic flat convergence for spacetimes that arise from a big bang and as the future maximal developments of initial data sets. For more information about intrinsic flat convergence see https://sites.google.com/site/intrinsicflatconvergence/

Speaker: Shing-Tung Yau (Tsinghua University; *remote via Zoom*)

Title: *Complete Noncompact Calabi-Yau Manifolds*

Abstract. Thanks to the pioneering work of Chern and Calabi, we see a burgeoning of complex differential geometry since 1940s. Nowadays, a very important theme in complex geometry is the Kähler-Einstein metric, which is the complex analogue of Einstein field equation. The Schwarzschild solution models the black hole – the most mysterious object in our universe. Actually, it is my fascination with finding such Ricci-flat metrics in general relativity that drove me to solve the Calabi conjecture in 1976. Just as singularities are inevitable in general relativity, it is important to study Calabi-Yau metric with singularities. I proposed a program to solve the noncompact version of the Calabi conjecture in the 1978 Helsinki International Congress of Mathematicians. In string theory, the extra dimensions of spacetime takes the form of a Calabi-Yau 3-fold. The noncompact version of Calabi-Yau is starting to show its significance in mirror symmetry. In the talk, I will survey Kähler-Einstein metric and especially the complete Ricci-flat Kähler metric on noncompact manifolds. The Yau-Tian-Zelditch expansion is one of the most important tools in its development.

Speaker: Yury Ustinovskiy (Lehigh University)

Title: *On Geometry of Steady Toric Kähler-Ricci Solitons*

Abstract. Let (M^{2n}, ω) be a Kahler manifold equipped with a Hamiltonian action of a half-dimensional torus T^n. I will explain how the fundamental equations of the Kahler geometry (Kahler-Ricci flat, Kahler-Einstein and Ricci solitons) reduce to real Monge-Ampere equations for a convex function on the dual of the Lie algebra of the torus: $Lie(T^n)^*$. In a particular case of toric gradient steady Kahler-Ricci solitons I will prove a rigidity result showing that the only complete solitons with a free T^n action are flat $(\mathbb{C}^*)^n$. The key ingredient in this proof will be the positivity of an appropriate Bakry-Emery Ricci tensor of the orbit space M^{2n}/T^n and a certain gradient estimate on such a background.

Speaker: Penny Smith (Lehigh University)

Title: *Quantum Geometry and Covariant Loop Quantum Gravity* (Part II)

Abstract. (You do not need to have attended Part I, given at this conference in 2019, to follow this talk, as the expository material will be repeated.)

In an attempt to quantize general relativity in a non-perturbational way, physicists have studied a discretization of its geometry and Action Functional called Regge Calculus. This is based on a piecewise flat triangularization of a space-time manifold with curvature concentrated at vertices. After much heuristic work by physicists, convergence of discrete geometric curvatures (Lipschitz-Killing curvatures) associated with a sequence of such triangularizations was proved in a brilliant 1984 paper of Jeff Cheeger.

However, as a method of quantizing general relativity the Regge calculus approach has–in common with most Quantum Field Theories–problems with infinities arising on small scales. This is very serious in general relativity, as general relativity is a non-renormalizable theory.

Another approach to quantizing general relativity in a non-perturbational way is Covariant Loop Quantum Gravity, which uses a triangularization (and a certain dual triangularization) with Lie Algebra and Lie Group representations associated to sub complexes. This is known as spin foam. Convergence of this model has been studied by physicists in very special cases using perturbations based on minimal uncertainty states. This is not satisfying because we are trying to study a non-perturbative model.

In Covariant Loop Quantum Gravity: lengths, areas, volumes, and dihedral angles of the triangularization are determined by the eigenvalues of left invariant vector fields satisfying an angular momentum algebra. Thus, there is a natural smallest length scale (corresponding to smallest non-zero eigenvalues) and no infinities arise from smaller scales. However, Space-Time should be a non-compact space, and so convergence of the triangularizations on large length scales is important.

This gives rise to a quantum geometry–for example, the metric geometry of a non-regular tetrahedron in three space is determined by six numbers (in Regge Calculus, by the lengths of its sides) in the LQG setting by the four face areas, its volume, and by a dihedral angle. All of which are quantum objects as described above.

We extend Cheeger's convergence method in the setting of LPG to provide a geometric analysis approach to convergence without resorting to perturbation methods.

In our talk, we describe LPG and its spin foams in the recent formulation of C. Rovelli, and sketch the above convergence. We also sketch a new quantitative convergence result for the above convergence.

Speaker: Herman Gluck (UPenn)

Title: *Homotopy Type of Spaces of Fibrations*

Abstract. What is the homotopy type of the space of all smooth fibrations of a given total space E by a given fibre F? Here are some results from the Penn Geometry-Topology group.

- **Question 1.** What is the homotopy type of the space of all smooth fibrations of a torus by oriented simple closed curves?

  Answer (Ziqi Fang). There are infinitely many components, one for each pair (p, q) of relatively prime integers, and each component is contractible.

- **Question 2.** What is the homotopy type of the space of all smooth fibrations of a 3-sphere by oriented simple closed curves?

  Answer (Everybody). There are just two components, and each component has the homotopy type of a 2-sphere.

- **Question 3.** What is the homotopy type of the space of all smooth fibrations of $S^2 \times S^1$ by oriented simple closed curves?

  Answer (Yi Wang and Jingye Yang). There are four components, and each component has the homotopy type of the based loop space ΩS^3 of the 3-sphere. This is the first example where the answer does not have the homotopy type of a finite CW-complex.

Speaker: Ernani Ribeiro Jr. (Universidade Federal do Ceará)

Title: *On Euler characteristic and Hitchin-Thorpe inequality for four-dimensional compact Ricci solitons*

Abstract. In this talk, we will discuss the geometry of 4-dimensional compact gradient Ricci solitons. We will show that, under an upper bound condition on the range of the potential function, a 4-dimensional compact gradient Ricci soliton must satisfy the classical Hitchin-Thorpe inequality. In addition, some volume estimates will be presented. This is a joint work with Xu Cheng and Detang Zhou.

Speaker: Claude LeBrun (Stony Brook)

Title: *Geometry of 4-Manifolds: Curvature in the Balance*

Abstract. This talk will focus on the Weyl functional on smooth compact 4-manifolds — i.e. the Riemannian curvature functional which sends each metric to the L^2-norm-squared of its conformal curvature. On Kaehler metrics, the Weyl functional is expressible in terms of the L^2-norm of the scalar curvature. However, the interaction between Weyl functional and the scalar curvature is far more subtle and indirect in the general Riemannian setting. The purpose of the talk will be to describe recent results and open questions regarding this relationship.

Speaker: Huai-Dong Cao (Lehigh University)

Title: *Geometry of Ricci Solitons*

Abstract. Ricci solitons are self-similar solutions to Hamilton's Ricci flow and a natural extension of Einstein manifolds. They often occur as singularity models in the Ricci flow. In this talk, I shall present some recent progress on Ricci solitons, including curvature estimates for expanding Ricci solitons and linear stability of compact shrinking Ricci solitons.

Speaker: Panagiota Daskalopoulos (Columbia University)

Title: *Ancient Solutions to Geometric Flows*

Abstract. Some of the most important problems in PDE are related to the understanding of singularities. This usually happens through a blow up procedure near the potential singularity which uses the scaling properties of the equation. In the case of a parabolic equation the blow up analysis often leads to special solutions which are defined for all time $-\infty < t \leq T$, for some $T \leq +\infty$. We refer to them as *ancient* solutions.

In this lecture we will discuss **Uniqueness Theorems** for ancient compact solutions to the *Ricci flow* and *Mean curvature flow.*